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or

$$z^2 + (x + a \cos \alpha)^2 = a^2,$$

$$x = \sqrt{a^2 - z^2} - a \cos \alpha.$$

The element of length of the arc is  $ds = adz/\sqrt{a^2 - z^2}$ , and the required surface

$$\begin{aligned} S &= \int 2\pi x ds = 2\pi \int_0^{a \sin \alpha} (\sqrt{a^2 - z^2} - a \cos \alpha) \frac{adz}{\sqrt{a^2 - z^2}} \\ &= 2\pi a \left[ z - a \cos \alpha \int \frac{dz}{\sqrt{a^2 - z^2}} \right]_0^{a \sin \alpha} \\ &= 2\pi a^2 \left[ \sin \alpha - \cos \alpha \left( \sin^{-1} \frac{z}{a} \right)_0^{a \sin \alpha} \right] = 2\pi a^2 (\sin \alpha - \alpha \cos \alpha). \end{aligned}$$

If  $\alpha = \pi/2$ ,  $S = 2\pi a^2$ , as it should be.

The object of the choice of coördinate axes as assigned in the statement of the problem is not evident.

The volume is

$$\begin{aligned} V &= \pi \int x^2 dz = \pi \int_0^{a \sin \alpha} (\sqrt{a^2 - z^2} - a \cos \alpha)^2 dz \\ &= \pi \left[ a^2(1 + \cos^2 \alpha)z - \frac{2}{3}z^3 - 2a \cos \alpha \left( \frac{z}{2} \sqrt{a^2 - z^2} + \frac{a^2}{2} \sin^{-1} \frac{z}{a} \right) \right]_0^{a \sin \alpha} \\ &= \pi a \left\{ \frac{1}{3}(2a^2 + a^2 \cos^2 \alpha) \sin \alpha - a \cos \alpha \cdot a \right\} \\ &= \pi a \left( \frac{2a^2 + c^2}{3} \sin \alpha - a \cos \alpha \right), \quad c = a \cos \alpha. \end{aligned}$$

If  $c = 0$ ,  $V = \frac{2}{3}\pi a^3$ , as it should be.

#### 431 (Calculus). Proposed by J. W. LASLEY, University of North Carolina.

Explain Bertrand's fallacy:

$$\begin{aligned} \int_{x=0}^{x=1} \int_{y=0}^{y=1} \frac{x^2 - y^2}{(x^2 + y^2)^2} dy dx &= \int_{y=0}^{y=1} \int_{x=0}^{x=1} \frac{x^2 - y^2}{(x^2 + y^2)^2} dx dy, \\ \frac{1}{4}\pi &= -\frac{1}{4}\pi, \quad 1 = -1. \end{aligned}$$

#### SOLUTION BY HORACE OLSON, Chicago, Illinois.

There is a theorem that the order of integration of a double integral may be reversed *if the integrand is continuous in both variables within the region of integration*. The integrand in this fallacy is discontinuous at  $(x = 0, y = 0)$ . Therefore, the theorem does not justify the reversal of the order of integration.

#### 341 (Mechanics). Proposed by PAUL CAPRON, U. S. Naval Academy.

A pole  $l$  feet long, with one end on the ground, touches the top of a wall  $a$  feet high and slides in a vertical plane perpendicular to the wall. Show that its instantaneous center of rotation is at the intersection of the vertical where it touches the ground with the perpendicular to its axis where it touches the wall, and that the locus of this center is a parabola having the latus rectum  $a$ .

#### SOLUTION BY S. W. REAVES, University of Oklahoma.

Let  $T$  be the point at the top of the wall and  $G$  the point on the ground through which the pole passes at some given instant. Let  $O$  be the point on the ground in the same vertical line with  $T$ , and let  $\theta$  be the angle  $TGO$ .

The direction of motion of any point is clearly at right angles to the line joining that point to the instantaneous center of rotation. (See Ziwet, *Theoretical Mechanics*, Art. 23; Demartres, *Cours de Géométrie infinitésimale*, Art. 20.) Hence, if the direction of motion of a point be known, the instantaneous center  $C$  must lie on the normal at the point to the direction of motion.

Now the direction of motion of that point of the rod which at the instant coincides with  $T$  is clearly along the rod. Hence the instantaneous center  $C$  is on the normal to the rod at  $T$ . Again, the direction of motion of the point  $G$  on the ground is along the ground. Hence,  $C$  is in the vertical line through  $G$ .

To find the locus of  $C$ , we choose  $OG$  and  $OT$  as coördinate axes. Then,

$$x = OG = OT \cot \theta = a \cot \theta \text{ and } y = GC = TG \csc \theta = a \csc^2 \theta.$$

Eliminating  $\theta$  between these two equations, we readily obtain

$$x^2 = a(y - a),$$

which is the equation of a parabola with its vertex at  $T$  ( $0, a$ ), its axis vertical, and latus rectum equal to  $a$ . The pole being of limited length  $l$ , the locus of  $C$  is that part of the parabola for which  $0 < x \leq \sqrt{l^2 - a^2}$ .

Also solved by H. R. HOWARD, WILLIAM HOOVER, J. B. REYNOLDS, and the PROPOSER.

### 342 (Mechanics). Proposed by WILLIAM HOOVER, Columbus, Ohio.

A uniform rod of length  $2a$  is freely hinged at one end, at the other end a string of length  $b$  is attached which is fastened at its further end to a point on the surface of a homogeneous sphere of radius  $c$ . If the masses of the rod and sphere are equal, find the motion of the system when slightly disturbed from the vertical, and the cubic equation giving the corresponding small oscillations.

#### SOLUTION BY THE PROPOSER.

For symmetry of notation, let there be three bodies of masses  $m_1, m_2, m_3$ ; of axes of symmetry whose lengths are  $2a_1, 2a_2, 2a_3$  with centers of gravity  $G_1, G_2, G_3$  distant  $a_1, a_2, a_3$  from corresponding extremities; radii of gyration about  $G_1, G_2, G_3$  equal to  $k_1, k_2, k_3$ ; the origin of rectangular coördinates at the fixed extremity of the highest body,  $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ , the horizontal through the fixed point being the axis of  $x$ ; and let  $\varphi_1, \varphi_2, \varphi_3$  be the angles which the axes of symmetry of the bodies make with the vertical at any time  $t$  from the beginning of motion. Then,

$$x_1 = a_1 \sin \varphi_1, \quad y_1 = a_1 \cos \varphi_1, \quad (1)$$

$$x_2 = 2a_1 \sin \varphi_1 + a_2 \sin \varphi_2, \quad y_2 = 2a_1 \cos \varphi_1 + a_2 \cos \varphi_2, \quad (2)$$

$$x_3 = 2a_1 \sin \varphi_1 + 2a_2 \sin \varphi_2 + a_3 \sin \varphi_3, \quad y_3 = 2a_1 \cos \varphi_1 + 2a_2 \cos \varphi_2 + a_3 \cos \varphi_3. \quad (3)$$

The kinetic potential equation is

$$\begin{aligned} T &= \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2 + k_1^2\dot{\varphi}_1^2) + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2 + k_2^2\dot{\varphi}_2^2) + \frac{1}{2}m_3(\dot{x}_3^2 + \dot{y}_3^2 + k_3^2\dot{\varphi}_3^2) \\ &= m_1ga_1 \cos \varphi_1 + m_2g(2a_1 \cos \varphi_1 + a_2 \cos \varphi_2) \\ &\quad + m_3g(2a_1 \cos \varphi_1 + 2a_2 \cos \varphi_2 + a_3 \cos \varphi_3) = V. \end{aligned} \quad (4)$$

From (1), (2), (3),

$$\dot{x}_1 = a_1 \cos \varphi_1 \cdot \dot{\varphi}_1, \quad \dot{y}_1 = -a_1 \sin \varphi_1 \cdot \dot{\varphi}_1, \quad (5)$$

$$\dot{x}_2 = 2a_1 \cos \varphi_1 \cdot \dot{\varphi}_1 + a_2 \cos \varphi_2 \cdot \dot{\varphi}_2, \quad \dot{y}_2 = -2a_1 \sin \varphi_1 \cdot \dot{\varphi}_1 - a_2 \sin \varphi_2 \cdot \dot{\varphi}_2, \quad (6)$$

$$\dot{x}_3 = 2a_1 \cos \varphi_1 \cdot \dot{\varphi}_1 + 2a_2 \cos \varphi_2 \cdot \dot{\varphi}_2 + a_3 \cos \varphi_3 \cdot \dot{\varphi}_3,$$

$$\dot{y}_3 = -2a_1 \sin \varphi_1 \cdot \dot{\varphi}_1 - 2a_2 \sin \varphi_2 \cdot \dot{\varphi}_2 - a_3 \sin \varphi_3 \cdot \dot{\varphi}_3. \quad (7)$$

These in (4) give

$$\begin{aligned} T &= \frac{1}{2}m_1(a_1^2 + k_1^2)\dot{\varphi}_1^2 + \frac{1}{2}m_2\{4a_1^2\dot{\varphi}_1^2 + 4a_1a_2 \cos(\varphi_1 - \varphi_2)\dot{\varphi}_1\dot{\varphi}_2 + (a_2^2 + k_2^2)\dot{\varphi}_2^2\} \\ &\quad + \frac{1}{2}m_3\{4a_1^2\dot{\varphi}_1^2 + 4a_2^2\dot{\varphi}_2^2 + (a_3^2 + k_3^2)\dot{\varphi}_3^2 + 8a_1a_2 \cos(\varphi_1 - \varphi_2)\dot{\varphi}_1\dot{\varphi}_2 \\ &\quad + 4a_2a_3 \cos(\varphi_2 - \varphi_3)\dot{\varphi}_2\dot{\varphi}_3 + 4a_1a_3 \cos(\varphi_1 - \varphi_3)\dot{\varphi}_1\dot{\varphi}_3\} \\ &= m_1ga_1 \cos \varphi_1 + m_2g(2a_1 \cos \varphi_1 + a_2 \cos \varphi_2) \\ &\quad + m_3g(2a_1 \cos \varphi_1 + 2a_2 \cos \varphi_2 + a_3 \cos \varphi_3) = V. \end{aligned} \quad (8)$$